Stabilisation of the orientational glass phase by cubic anisotropy

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Abstract. Though mixed crystals can form orientational glasses, the short-range model of the isotropic quadrupolar glass does not have a glass phase in three dimensions at non-zero temperature. A Monte Carlo investigation is presented which suggests that the glass phase is stabilised by cubic anisotropy.

1. Introduction

There are mixed crystals that can form an orientational glass. Examples are $K(Br)_x(CN)_{1-x}$ (see Knorr 1987) and $(N_2)_x Ar_{1-x}$ (Barret and Meyer 1964, Press *et al* 1982).

These systems have been modelled as quadrupolar glasses with bond disorder (Goldbart and Sherrington 1985, Carmesin and Binder 1987). These authors considered a simple cubic lattice in d dimensions ($d = \infty$ in the mean-field case) with a uniaxial quadrupole on each site. Each uniaxial quadrupole is presented by a unit vector, s_i, that orients freely in m dimensions. The quadrupole tensor is

$$f_i^{\mu\nu} = s_i^{\mu} s_i^{\nu}. \tag{1.1}$$

Two particles interact with the scalar product of tensors:

$$H_{ij} = \sum_{\mu,\nu}^{m} f_i^{\mu\nu} f_j^{\mu\nu} = \left(\sum_{\mu}^{m} s_i^{\mu} s_j^{\mu} \right)^2. \tag{1.2}$$

The total energy is

$$H = \sum_{i<j}^{N} J_{ij} H_{ij}. \tag{1.3}$$

Here J_{ij} is distributed via a Gaussian with zero mean:

$$P(J_{ij}) = \frac{1}{(2\pi)^{1/2}} \exp(-\tfrac{1}{2} J_{ij}^2). \tag{1.4}$$

A Monte Carlo (MC) investigation of the isotropic quadrupolar glass with nearest-neighbour interaction suggests that the lower critical dimension is larger than three (Carmesin and Binder 1987). Presumably, the above isotropic model cannot describe the above-mentioned orientational glasses, $KBr_x(CN)_{1-x}$ and $Ar_x(N_2)_{1-x}$.

Here we present a MC investigation of the cubic anisotropic quadrupolar glass (CAQG) for $m = d = 3$. This model is defined by (1.1), (1.3) and (1.4) and by the cubic anisotropic two-particle interaction:

$$H_{ij} = \sum_{\mu\nu}^{m} f_i^{\mu\nu} f_j^{\mu\nu} \delta^{\mu\nu} = \sum_{\mu}^{m} (s_i^{\mu} s_j^{\mu})^2. \tag{1.5}$$

In the mixed crystal $K(Br)_x(CN)_{1-x}$ there is a cubic anisotropy because, below the freezing temperature of the translational degrees of freedom and above the freezing temperature of the orientational degrees of freedom, it forms the cubic NaCl structure. Below the freezing temperature of the orientational degrees of freedom, at least the topology of the NaCl structure is conserved.

The mixed crystal $(N_2)_x Ar_{1-x}$ forms a FCC or a HCP structure. Thus there exists a global anisotropy and, though this is not the cubic anisotropy, it might have similar effects for the orientational glass phase because both anisotropies are caused by the crystalline structure.

The simulations were performed on a VP 100 vector computer with the checkerboard algorithm. We obtained a speed of more than 0.5×10^6 updatings per second. The whole calculation took about 200 h of CPU time. At high temperatures simple cubic lattices of size 12^3 were investigated and 80–500 bond averages were taken. At lower temperatures we simulated a simple cubic lattice of size 18^3 and took 20 bond averages, having the same statistics because there are more bonds in a larger lattice.

As the correlation length was only of order two at all the temperatures we investigated, we did not make a finite-size analysis. We investigated the time-dependent glass susceptibility to be sure that we obtained the static value of the glass susceptibility.

2. Static properties

While the unit vectors are distributed isotropically in the isotropic quadrupolar glass, as long as there is no spontaneous breaking of the global symmetry of the Hamiltonian, the particles are not distributed isotropically in CAQG because the Hamiltonian itself breaks the global symmetry. Therefore we investigate the one-particle distribution function:

$$P(s) = (\langle\langle\langle \delta(s - s_i)\rangle_T\rangle_N\rangle_{av} \tag{2.1}$$

with

$$s = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad x^2 + y^2 + z^2 = 1 \tag{2.2}$$

where $\langle\ \rangle_T$ is the mean over the canonical ensemble, $\langle\ \rangle_N$ is the mean over the particles and $(\ )_{av}$ is the mean over the Gaussian distribution of the bonds.

We describe the one-particle distribution function by its moments. These are the expectation values of some products of the components of s:

$$\langle x^a y^b z^c \rangle_p = \int P(s) x^a y^b z^c \, ds \left(\int ds \right)^{-1}. \tag{2.3}$$

As the quadrupolar Hamiltonian is invariant after a rotation of all unit vectors through an angle π, the moments are also invariant after that rotation. Thus a, b and c in (2.3) are even for non-zero moments. We call the sum $a+b+c$ the order of the moment. We obtain three independent moments up to sixth order (Carmesin 1988b):

$$q_{41} = 5\langle x^4 + y^4 + z^4 \rangle_p - 3 \tag{2.4}$$

$$q_{61} = 7\langle x^6 + y^6 + z^6 \rangle_p - 3 \tag{2.5}$$

$$q_{62} = 105\langle x^2 y^2 z^2 \rangle_p - 1 \tag{2.6}$$

with

$$q_{41}(T = \infty) = q_{61}(T = \infty) = q_{62}(T = \infty) = 0. \tag{2.7}$$

If we require the unit vectors to be directed along the axes, our model, (1.1), (1.3)-(1.5), describes the Potts glass. For the Potts glass the three moments are

$$q_{41}^{\text{Potts}} = 2 \tag{2.8}$$

$$q_{61}^{\text{Potts}} = 4 \tag{2.9}$$

$$q_{62}^{\text{Potts}} = -1. \tag{2.10}$$

If the temperature is lowered, the one-particle distribution of CAQG approaches that of the Potts glass continuously; see figure 1. All data in figure 1 (except those at zero temperature) come from MC simulations which are taken from (2.8)-(2.10).

The mean-field free energy of CAQG contains a term of an order parameter due to anisotropy (Carmesin and Ohno 1988). This order parameter is exactly the leading moment of the one-particle distribution function. This order parameter has got one replica index only and thus it describes a global breaking of symmetry, i.e. the mean over the volume and the bonds of the mean over the canonical ensemble of x^4 is non-zero at low temperatures. Thus figure 1 is in agreement with mean-field theory.
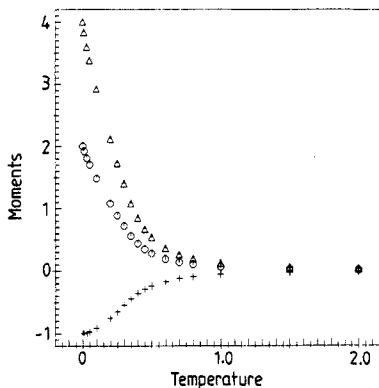

Figure 1. The three leading moments of the one-particle distribution function plotted against temperature. The circles show q_{41}, the triangles q_{61}, and the $+$ signs q_{62}. At high temperatures the quadrupoles are distributed nearly isotropically; they are mostly directed along the axes at low temperatures. At zero temperature the values for the Potts glass are plotted: $q_{41}^{\text{Potts}} = 2$, $q_{61}^{\text{Potts}} = 4$ and $q_{62}^{\text{Potts}} = -1$, while at all non-zero temperatures the Monte Carlo data for the cubic anisotropic quadrupolar glass are shown.

In order to equilibrate the system in the simulation, the system was cooled continuously to the temperature under consideration. Initially, all the orientations were chosen at random. The temperature was lowered according to the following equation:

$$\frac{1}{T(t)} = \frac{1}{T}\begin{cases} t/\tau^* & t \leqslant \tau^* \\ 1 & t \geqslant \tau^*. \end{cases} \tag{2.11}$$

The statistical averages are taken for times larger than τ^*.

At very low temperatures this procedure is not sufficient to obtain the equilibrium internal energy. We obtain approximations by an extrapolation that was first proposed by Grest *et al* (1986):

$$E(\tau) = E(\tau^* = \infty) + \text{constant} \times [1/\ln(\tau^*)]. \tag{2.12}$$

While in the Potts glass there is a point of inflection of the internal energy at the temperature 0.4 (figure 2) such a point could not be identified in the simulation of CAQG. In CAQG the energy at rather low temperatures rapidly decreases when the particles are preferentially oriented in the directions of the axes (see figures 1 and 2). CAQG lowers its energy more rapidly with falling temperature than the isotropic quadrupolar glass and the Potts glass.
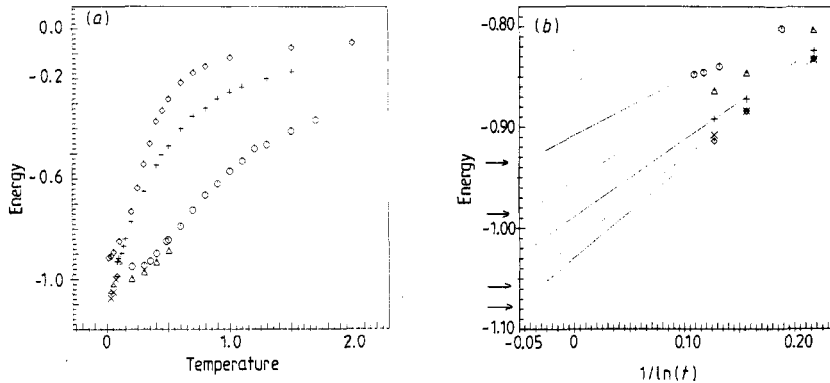

Figure 2. (*a*) The internal energy plotted against temperature for the cubic anisotropic quadrupolar glass, the isotropic quadrupolar glass (see Carmesin and Binder 1987), and the Potts glass (see Carmesin and Binder 1988). For low temperatures extrapolated values are also presented. The circles (large triangles) show the (extrapolated) energy of the Potts glass, the + signs (× signs) show the (extrapolated) energy of the isotropic quadrupolar glass, while the diamonds (small triangles) present the (extrapolated) energy of the cubic anisotropic quadrupolar glass. (*b*) For the case of the cubic anisotropic quadrupolar glass, the extrapolation of the internal energy for infinite time is demonstrated for several temperatures. The circles show the energy for $T = 0.2$, the triangles for $T = 0.8$, the + signs for $T = 0.05$, the × signs for $T = 0.03$ and the diamonds for $T = 0.01$. The arrows show the extrapolated energies.

We consider two types of correlations (for a systematic discussion see Carmesin (1988a)). The total isotropic correlation was considered in the isotropic quadrupolar glass (see Carmesin and Binder 1987). It is the isotropic two-particle interaction subtracted by its value at infinite temperature:

$$\text{cor}_{ij}^t = \left(\sum_{\mu}^{m} s_i^{\mu} s_j^{\mu}\right)^2 - \frac{1}{m}. \tag{2.13}$$

The diagonal correlation is the cubic anisotropic correlation, subtracted by its value at infinite temperature:

$$\text{cor}_{ij}^{d} = \sum_{\mu}^{m} (s_i^{\mu} s_j^{\mu})^2 - \frac{1}{m}. \tag{2.14}$$

From each such correlation, we define the following correlation functions. The spatial correlation function is

$$g(r) = C_0(\langle\langle \text{cor}_{ij}\rangle_T^2\rangle_{|r_i - r_j| = r})_{\text{av}}. \tag{2.15}$$

Here $\langle \rangle_{|r_i - r_j| = r}$ describes the mean over those pairs of particles that are separated by a distance r. We normalise this function such that for completely frozen, but isotropically distributed, particles it is one (Carmesin and Ohno 1988). Thus we obtain

$$C_0^d = \frac{225}{8} \qquad C_0^t = \frac{45}{4}.$$

From the correlation function we obtain the quadratic susceptibility, which is the generalisation of the spin-glass susceptibility:

$$\chi_G = \int dr^d \, g(r). \tag{2.16}$$

The overlap of two configurations a and b is

$$Q(a, b) = \frac{m}{m-1} (\langle \text{cor}(s_i(a), s_i(b)) \rangle_N)_{\text{av}}. \tag{2.17}$$

In the MC simulation we investigate overlaps for total isotropic correlations (2.13) and for cubic anisotropic correlations (2.14) which are inserted for cor in the above equation. From this, the autocorrelation function and the glass order parameter are obtained:

$$q(t) = \frac{1}{t_{\text{obs}}} \int_0^{t_{\text{obs}}} Q(t', t' + t) \, dt' \tag{2.18}$$

$$q_{\text{EA}} = \lim_{t \to \infty} q(t). \tag{2.19}$$

The quadratic susceptibility for diagonal correlations as a function of temperature shows curvature on a log-log plot against temperature, and thus we are either above the scaling region and the lower critical dimension, d_1, is above three (where for a zero-temperature transition one must get a straight line on such a log-log plot) or $d_1 \lesssim 3$, figure 3(a).

If the system is at d_1, we expect that $\ln \chi \propto T^{-\gamma'}$, $\ln \xi \propto T^{-\gamma'}$ and $\ln \tau \propto T^{-z'\nu'}$ with $\gamma' = 2$ and $z'\nu' = 3$ in the Ising spin glass (McMillan 1984). The susceptibility fits a straight line on a $\ln(\ln \chi)$ against $\ln T$ plot quite nicely, but $\gamma' = 2.3$ rather than $\gamma' = 2$ suggests that the system might be above d_1, figure 3(b). We estimate the error bars by a very crude method: we fit a reasonable straight line with maximum slope and another one with minimum slope to the data points in figures 7(g) and 3(b). From these we obtain $\gamma' = 2.3 \pm 0.3$ and $z'\nu' = 3.5 \pm 0.4$. Figures 3(c) and (d) show plots of the glass susceptibility against $1 - 0.1/T$ and $1 - 0.2/T$ respectively, and yield $\gamma = 15.21$ and 4.35 respectively. While $\gamma = 15.21$ is likely to be too large, $\gamma = 4.35$ is reasonable.
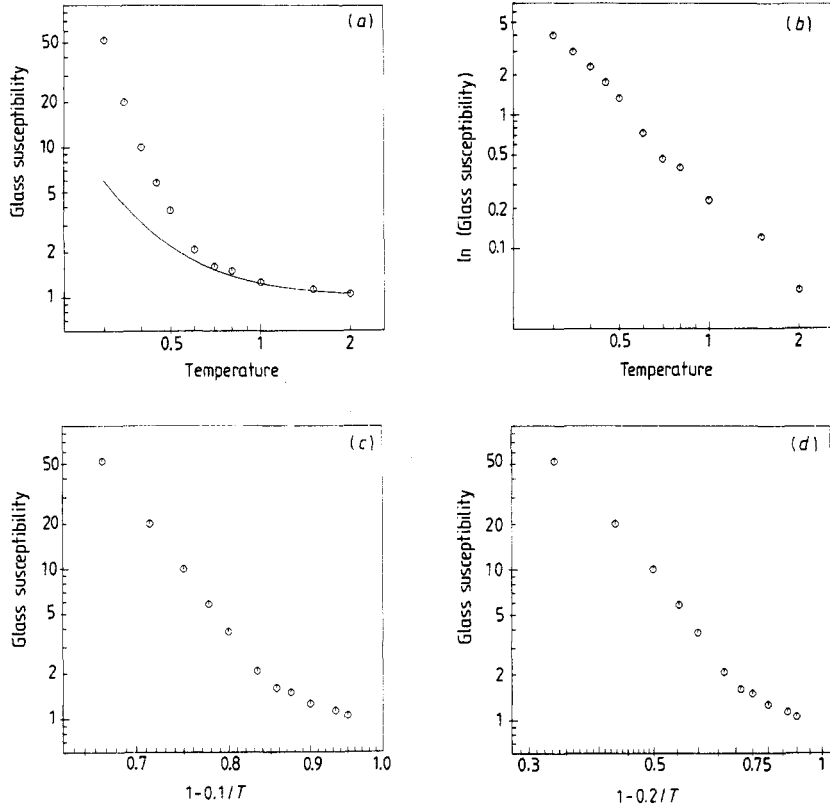
Figure 3. (*a*) The glass susceptibility for diagonal correlations is shown as a function of temperature. The full curve is the result of a high-temperature expansion up to the order T^{-4} (Carmesin and Ohno 1988). The MC data agree with the expansion above the temperature $T = 0.7$, while below that temperature the expansion is much too small because higher-order terms are missing. One also has to keep in mind that it is still an open question whether the expansion converges at all (Bray 1987). (*b*) The logarithm of the glass susceptibility for diagonal correlations plotted against temperature. A linear curve in the above double logarithmic scale is consistent with the assumption that the critical dimension is three. (*c*) The glass susceptibility for diagonal correlations plotted against $1 - 0.1/T$. (*d*) The glass susceptibility for diagonal correlations plotted against $1 - 0.2/T$.

From the correlation function for diagonal correlations, figure 4(*a*), the correlation length is obtained by a fit to

$$g(r) \approx \exp(-r/\xi). \tag{2.20}$$

In figures 4(*b*) and (*c*) ξ is plotted against $1 - 0.1/T$ and $1 - 0.2/T$ respectively. This yields $\nu = 7.2$ and 1.14 respectively. As $\nu = 7.2$ is likely to be too large, while $\nu = 1.14$ is reasonable, the correlation length as well as the glass susceptibility support the assumption of a non-zero transition to the glass phase at $T_c \approx 0.2$.

In order to investigate the linear response of the system, an external field, *h*, is applied which directs the particles along the *z* axis. The conjugated order parameter is zero at infinite temperature and thus is

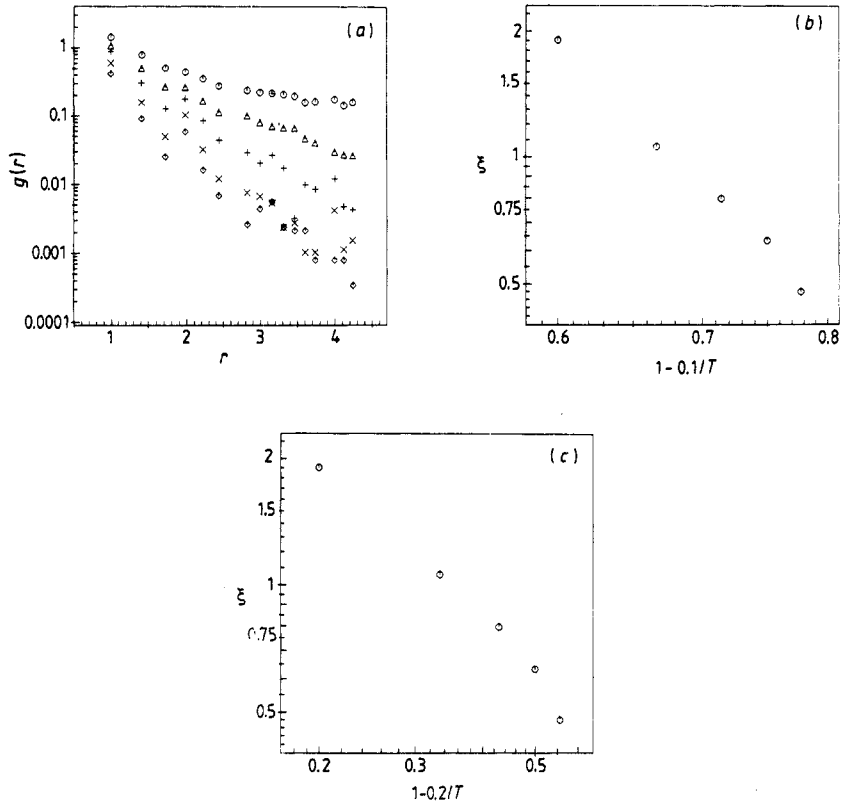$$o = \sum_i^N (s_i^z)^2 - \tfrac{1}{3}. \tag{2.21}$$

Figure 4. (*a*) The correlation function for diagonal correlations plotted for the temperatures $T = 0.25$ (circles), $T = 0.3$ (triangles), $T = 0.35$ (+ signs), $T = 0.4$ (× signs) and $T = 0.45$ (diamonds) against the distance. At these short distances, the anisotropy of the lattice is still seen. At distance 2, for example, the values are relatively large compared to the distance $\sqrt{3}$, as only two bonds connect particles with distance 2 while three bonds connect particles of distance $\sqrt{3}$. The correlation function decays nearly exponentially. (*b*) The correlation length for the diagonal correlations plotted against $1 - 0.1/T$. (*c*) The correlation length for the diagonal correlations plotted against $1 - 0.2/T$.

The Hamiltonian thus contains the energy due to the applied field:

$$H = \sum_{i<j}^{N} J_{ij} H_{ij} - ho. \tag{2.22}$$

The conjugated susceptibility is

$$\chi_{\text{lin}} = \lim_{h \to 0} (1/h)[\langle o \rangle_T]_{\text{av}}. \tag{2.23}$$

In the simulation, a field of strength 0.1 is applied. The linear response is approximately proportional to $1/T$ at high temperatures, while it 'saturates' near $T = 0.2$; see figure 5. At high temperatures the particles move almost uncorrelated and thus the linear susceptibility is proprtional to $1/T$. At low temperatures the particles are correlated due to the onset of disordered correlations among the particles. These correlations are given by the glass susceptibility, figure 3(*a*), which indeed seems to have a singularity near $T_c \approx 0.2$.
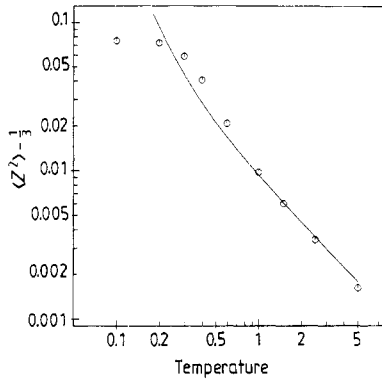
Figure 5. The order parameter describing the directing of particles along the z axis, equation (2.32), is plotted against temperature. The applied field is $\frac{1}{10}$. While at high temperature the response is approximately proportional to $1/T$, it 'saturates' near $T = 0.2$. The full curve shows the result of the high-temperature expansion up to T^{-3} (Carmesin and Ohno 1988).

3. Dynamics

We consider the following decay laws for the autocorrelation functions for diagonal and total correlations.

(i) Logarithmic decay:

$$q(t) \propto \log(t). \tag{3.1}$$

(ii) Algebraic decay:

$$q(t) \propto t^{-a}. \tag{3.2}$$

(iii) Stretched exponential decay:

$$q(t) \propto \exp[-(t/\tau')^{y}]. \tag{3.3}$$

(iv) Enhanced power law (van Hemmen and Sütö 1985):

$$q(t) \propto \exp[-A \ln(t/\tau'')^{y''}]. \tag{3.4}$$

The autocorrelation function of diagonal correlations does not fit any of the first three considered decay laws, figures 6(a), (b) and (c). As there is curvature to the upper left for large times in figure 6(c), the enhanced power law can be excluded as well.

The autocorrelation function of total isotropic correlations fits neither to logarithmic decay, figure 7(a), nor to algebraic decay, figure 7(b), but does fit to the stretched exponential at intermediate and late times, figure 7(c).

Therefore, we extract the parameters y and τ' from the data. We integrate the autocorrelation time, τ, from the parameters obtained above:

$$\tau = \int_{0}^{\infty} q(t) \, dt \tag{3.5}$$

$$= \int_{0}^{\infty} \exp[-(t/\tau')^{y}] \, dt \tag{3.6}$$

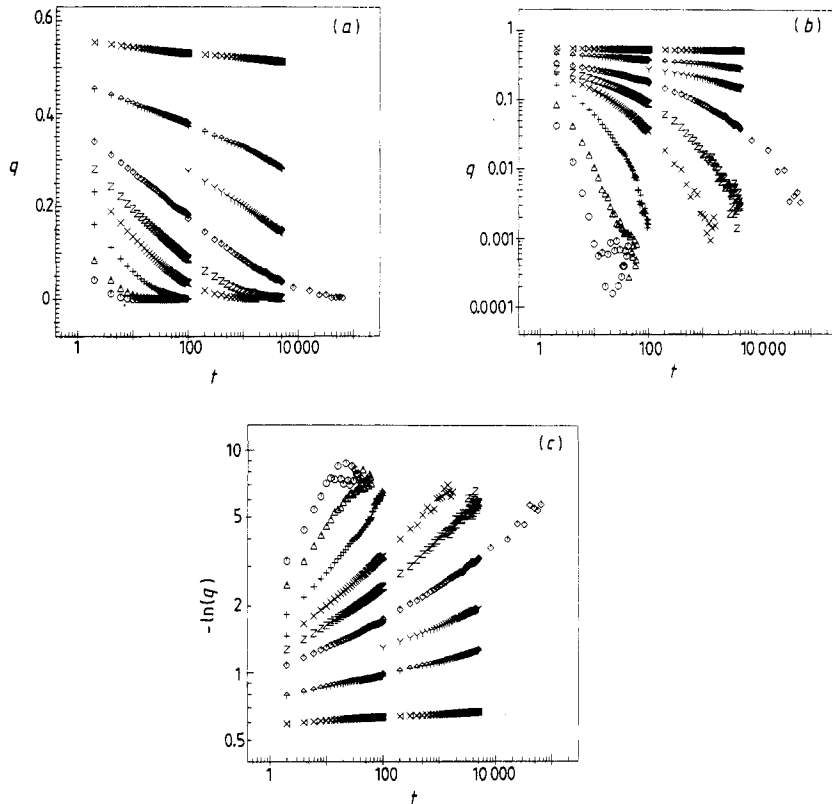$$= (\tau'/y)\Gamma(1/y) \xrightarrow[y \to 0]{} (\tau'/y) \exp[(1/y) \ln(1/y)^{y}]. \tag{3.7}$$

Figure 6. The autocorrelation function for diagonal correlations is plotted against time for several temperatures, for (a), logarithmic decay, (b) algebraic decay, and (c) stretched exponential decay. A straight line is expected in each case. In the figures $6(a)$-(c) and $7(a)$-(c) the autocorrelation functions are plotted for the temperatures $T = 1.0$ (circles), $T = 0.7$ (triangles), $T = 0.5$ (+ signs), $T = 0.4$ (× signs), $T = 0.35$ (Z signs), $T = 0.3$ (diamonds), $T = 0.25$ (Y signs), $T = 0.2$ (arrows) and $T = 0.1$ (⋈ signs).

If y vanishes, then τ must diverge. The function y is a linear function of temperature over a wide range; see figure $7(d)$. If we extrapolate $y(T)$ to zero, we find that y vanishes at $T = 0.08$. Thus τ must diverge at $T = 0.8$ and therefore $T = 0.08$ is in the glass phase. Therefore consideration of y suggests that $d_1 < 3$ and $T_c > 0.08$. We do not give an estimate for the error bars here, but we show in table 1 that the size of $y(T)$ is larger for the isotropic quadrupolar glass than for CAQG.

Of course, τ can diverge at higher temperatures than 0.08, as is obvious from (3.7). Therefore we consider τ as a function of $1 - 0.1/T$ and $1 - 0.2/T$ in figures $7(e)$ and (f) respectively, which yields $zv = 66$ and 15.9 respectively. As $zv = 66$ is very large for a dynamical exponent, $T_c \approx 0.2$ is more likely. We consider $d_1 = 3$ in figure $7(g)$. We find $z'v' = 3.5 \pm 0.4$, which is larger than the value of 3 that McMillan (1984) predicted for the Ising model, and thus this plot also suggests $d_1 < 3$. For the case $d_1 > 3$, McMillan predicted an Arrhenius law:

$$\tau \propto \exp(E/T). \tag{3.8}$$

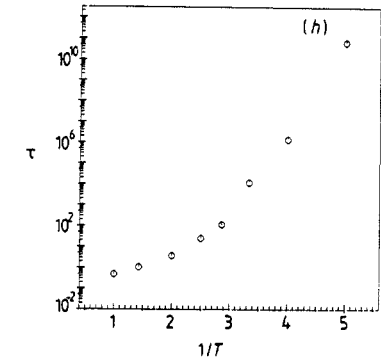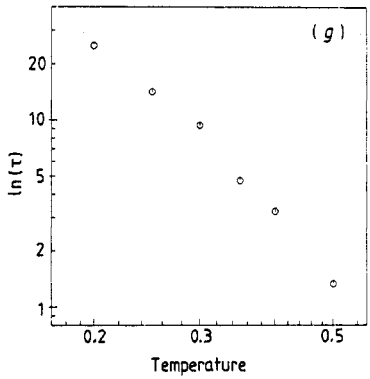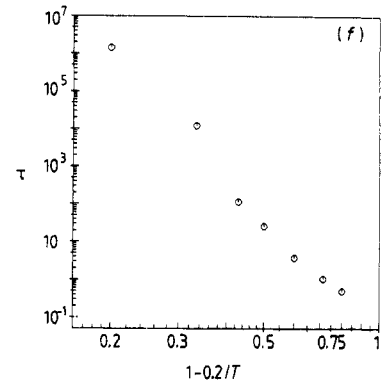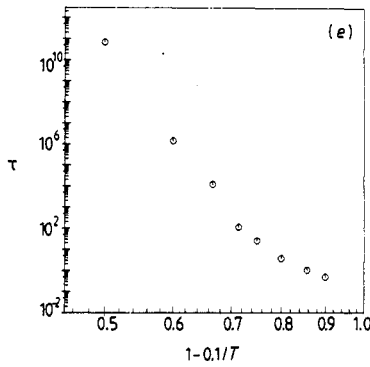Figure $7(h)$ shows that this does not at all fit the data.
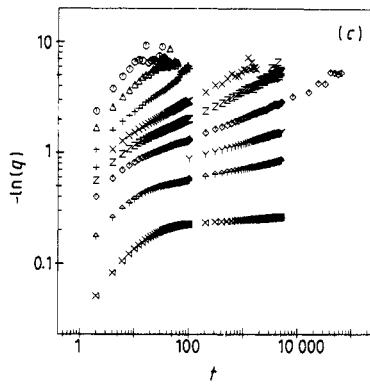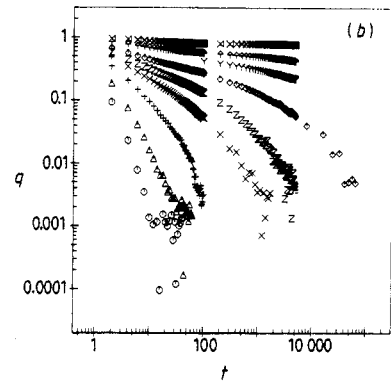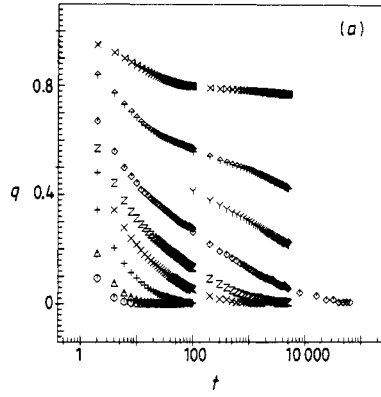
Table 1.

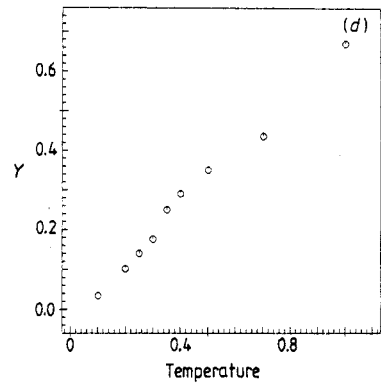| T	$\tau_{\text{isotropic}}$	$\tau_{\text{cubic anisotropic}}$	$y_{\text{isotropic}}$	$y_{\text{cubic anisotropic}}$
1.5	1			
1	1.23			0.67
0.8	1.71			
0.7		1.07		0.436
0.6	2.02		0.57	
0.5	5.44	3.78	0.52	0.35
0.4	12.51	25.7	0.483	0.29
0.35		114.2		0.25
0.3		12 075.0		0.176
0.25	101.4	1.427×10^6 (extrapolated slightly)	0.43	0.14
0.2	345.41	6.77×10^{10} (extrapolated strongly)	0.41	0.102
0.15	1 660.0		0.385	
0.13	3 603.0		0.375	
0.11	10 910.0		0.36	
0.1				0.034
0.09	53 657.0			

In order to compare CAQG with isotropic quadrupolar glass without any extrapolation, we present absolute values of the glass susceptibility and τ for both glasses. One should recognise that the absolute strength of the interaction, which is measured by the variance of the two-particle interaction at infinite temperature, $\langle (H_{ij} - \langle H_{ij} \rangle_{T=\infty})^2 \rangle_{T=\infty}$, is larger for the isotropic quadrupolar glass $[2(m-1)/m^2(m+1)]$ than for CAQG $[4(m-1)/m^2(m+2)^2]$. From this one would naively expect that τ is larger for the isotropic quadrupolar glass than for CAQG at the same temperature.

4. Conclusions

We saw that the data obtained from the MC simulation do suggest that there is a transition to the glass phase in three dimensions. But now we raise the question of how reliable this observed 'trend' really is.

We first have to admit that a MC simulation can, in no case, yield a rigorous proof of the stability of any phase, because it is necessary to extrapolate the obtained data.

Figure 7. The autocorrelation function for total isotropic correlations plotted against time for several temperatures (see caption of figure 6 for explanation of symbols in (a)-(c)). (a) For logarithmic decay a straight line is expected. (b) For algebraic decay a straight line is expected. (c) A straight line occurs for a stretched exponential decay. (d) The exponent in the stretched exponential decay law, extracted from figure 7(c), is plotted against temperature. (e) The autocorrelation time, obtained from equation (3.7), plotted against $1 - 0.1/T$. (f) The autocorrelation time, obtained from equation (3.7), plotted against $1 - 0.2/T$. (g) The logarithm of the autocorrelation time, obtained from equation (3.7), plotted against temperature. (h) The logarithm of the autocorrelation time, obtained from equation (3.7), plotted against the inverse temperature.

On the other hand, even in the case of the Ising spin glass, in three dimensions T_c can also only be obtained by methods which have to extrapolate the obtained data (Binder and Young 1986). We therefore raise three questions.

(i) How reliable is the estimate of the lower critical dimension? We do not exclude that d_l is three, though the data suggest that it is lower. The data suggest more strongly that d_l is not larger than three, since if that was the case the critical exponents would be even larger than those we obtain for the assumed $T_c = 0.1$, $z\nu = 66$ and $\gamma = 15.21$. Additionally, the Arrhenius law does not fit τ.

Thus, from extrapolation of MC data, we conclude $d_l \leqslant 3$. If so, the quadrupolar glass phase is stabilised by cubic anisotropy, because from extrapolation of MC data obtained for the isotropic quadrupolar glass (Carmesin and Binder 1987, Carmesin 1988b) we conclude that $d_l > 3$.

(ii) How reliable are the numerical estimates of T_l and the critical exponents? From the obtained MC data we do not dare to make an estimate of the value of T_l. We only investigate the approximate critical exponents that the candidates of critical temperatures $T_c = 0.1$ and $T_c = 0.2$ would yield. Since a lower T_l would yield larger critical exponents, and additional MC data at lower temperatures than those investigated here would lead to larger critical exponents as well, we conclude that the approximate critical exponents that we obtain here are: (a) *lower bounds* for critical exponents, obtained at the suggested transition temperature, (b) *lower bounds* for the critical exponents for a transition at zero temperature, if d_l is larger than three, and (c) *rough estimates* of critical exponents, yielding the order of magnitude of the exponent for a given T_c.

(iii) What can we conclude from the MC data without any extrapolation? (a) We conclude that the available phase space shrinks dramatically with decreasing temperature (figure 1). (b) The specific heat is large compared to that of the isotropic quadrupolar glass and that of the Potts glass (slopes in figure 2(a)). (c) The absolute values of τ are larger than the values obtained for the isotropic quadrupolar glass,

Table 2.

T	$\chi_{\text{isotropic}}$	$\chi_{\text{cubic anisotropic}}$
2.0		1.05
1.5	1.21	1.13
1.2	1.37	
1	1.6	1.26
0.8		1.5
0.7	2.26	1.6
0.6	2.35	2.08
0.5	3.3	3.8
0.45	4.19	5.8
0.4		10.0
0.35		20
0.3	9.46	52.1
0.27	10.11	
0.25	13.7	
0.2	22	
0.15	33	
0.13	79.5	
0.09	194.1	

though the variance of the two-particle interaction, $\langle (H_{ij} - \langle H_{ij} \rangle_{T=\infty})^2 \rangle_{T=\infty}$, is $2(m-1)/m^2(m+1)$ for isotropic interactions, but only $4(m-1)/m^2(m+2)^2$ for cubic anisotropic interactions (table 1). (d) The absolute values of the quadratic susceptibility, which measures finite correlations above a possible glass transition temperature, are larger than those in the isotropic quadrupolar glass (table 2). (e) The absolute values of the phenomenological exponent $y(T)$ are larger in the isotropic quadrupolar glass than in CAQG. (f) The autocorrelation times grow stronger than predicted by an Arrhenius law.

In principle, there are two glass transitions possible in CAQG (Carmesin 1988a). At the temperature at which the diagonal correlations turn from zero to non-zero, one transition temperature is defined. At the temperature at which the non-diagonal correlations, i.e. the difference between the total isotropic and diagonal correlations, turn from zero to non-zero, another transition temperature is defined. In the analysis of the MC data it is assumed that these two temperatures cannot be distinguished.

In vector spin glasses, for comparison, the glass phase is probably stabilised by the Dzyaloshinskii–Moriya anisotropy (Bray and Moore 1987, Kotliar 1987).

Acknowledgments

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